SM1 Chapter 6A Practice Test (6.0-6.4)

Exponential Functions

Name_____

Period _____ Date_____

Remember, x is what you plug in for the exponent. Follow ORDER OF OPERATIONS carefully

Evaluate each function for the given value.

1) $f(x) = 3 \cdot 2^x$ for $x = -2$

$3 \cdot 2^{-2}$ $\quad \frac{3}{1} \cdot \frac{1}{4}$ $\quad \boxed{\frac{3}{4}}$

$3 \cdot \left(\frac{1}{2}\right)^2$

2) $f(x) = 3^x + 1$ for $x = 0$

$3^0 + 1$ $\quad \boxed{2}$

$1 + 1$

3) $f(x) = \left(\frac{1}{3}\right)^x$ for $x = 3$

$\left(\frac{1}{3}\right)^3 = \frac{1^3}{3^3} = \boxed{\frac{1}{27}}$

$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$

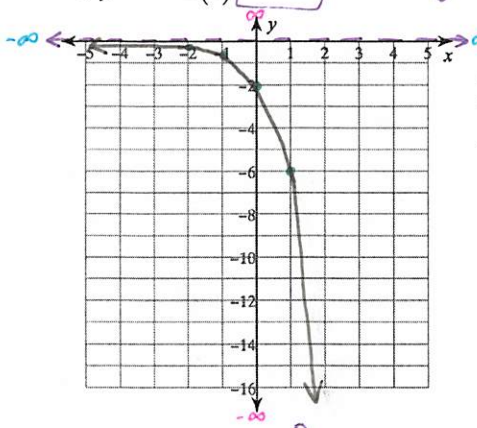
4) $f(x) = \left(\frac{1}{2}\right)^x$ for $x = -3$

$\left(\frac{1}{2}\right)^{-3} \rightarrow \left(\frac{2}{1}\right)^3$ $\quad \boxed{8}$

$2 \cdot 2 \cdot 2$

Sketch the Graph of the function and then state the Horizontal Asymptote, Domain and Range.

5) $y = -2(3)^x$ [ ] ← nothing there (+0)



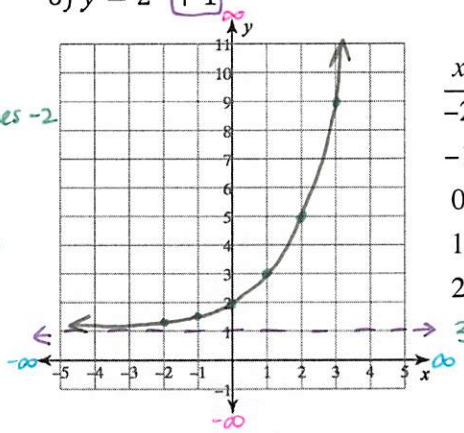
x	y	
−2	−0.22 (or $-\frac{2}{9}$)	do 3^{-2}, then times −2
−1	−0.66 (or $-\frac{2}{3}$)	
0	−2	$(-2 \cdot 3^0 = -2 \cdot 1)$
1	−6	$(-2 \cdot 3^1 = -2 \cdot 3)$
2	−18	$(-2 \cdot 3^2 = -2 \cdot 9)$

Asymptote: $y =$ 0
(where the curve will approach but not cross)

Domain: $(-\infty, \infty)$ or All real numbers
(x-values covered)

Range: $(-\infty, 0)$ or $y < 0$
(y-values covered; same value as asymptote)

6) $y = 2^x + 1$



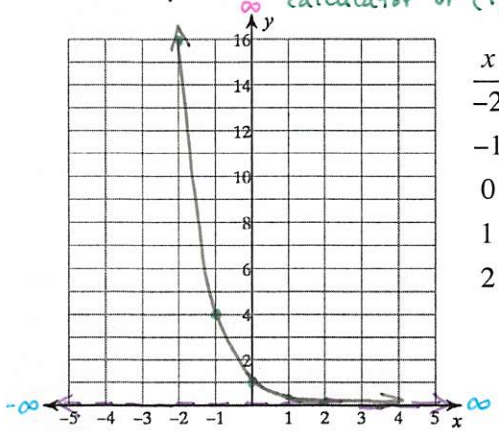
x	y	
−2	1.25	$(2^{-2}+1 = \frac{1}{4}+1)$
−1	1.5	$(2^{-1}+1 = \frac{1}{2}+1)$
0	2	$(2^0+1 = 1+1)$
1	3	$(2^1+1 = 2+1)$
2	5	$(2^2+1 = 4+1)$
3	9	$(2^3+1 = 8+1)$

Asymptote: $y =$ 1

Domain: $(-\infty, \infty)$ or All real #s

Range: $(1, \infty)$ or $y > 1$

7) $y = \left(\frac{1}{4}\right)^x$ ← you can use 0.25 in calculator or (1/4)



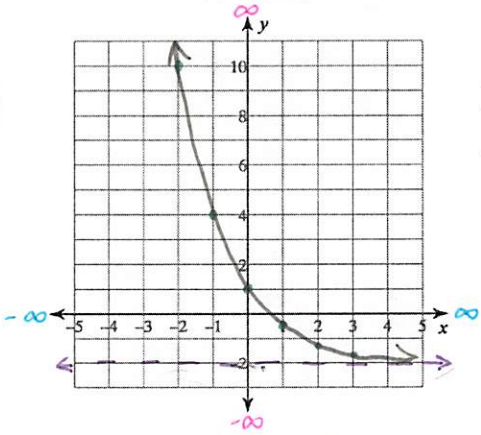
x	y	
−2	16	$(\frac{1}{4}^{-2}$ or $0.25^{-2})$
−1	4	$(\frac{1}{4}^{-1})$
0	1	$(\frac{1}{4}^{0})$
1	0.25	$(\frac{1}{4}^{1} = \frac{1}{4})$
2	0.0625	$(\frac{1}{4}^{2} = \frac{1}{16})$

Asymptote: $y =$ 0

Domain: $(-\infty, \infty)$ or all real #s

Range: $(0, \infty)$ or $y > 0$

8) $y = 3\left(\frac{1}{2}\right)^x$ [−2] ← or 0.5



x	y	
−2	10	$(3 \cdot (\frac{1}{2})^{-2} - 2 = 3 \cdot 4 - 2)$
−1	4	$(3 \cdot (\frac{1}{2})^{-1} - 2 = 3 \cdot 2 - 2)$
0	1	$(3(\frac{1}{2})^0 - 2 = 3 \cdot 1 - 2)$
1	−0.5	
2	−1.25	
3	−1.625	$(3(\frac{1}{2})^3 - 2 = 3(\frac{1}{8}) - 2)$

Asymptote: $y =$ −2

Domain: $(-\infty, \infty)$ or all real #s

Range: $(-2, \infty)$ or $y > -2$

Determine whether each of the following represents an exponential function. (Check for constant ratio) (or multiplication pattern)
If it is an exponential function, state whether it is exponential growth or decay. Explain.

9)

x	-1	0	1	2
y	1	4	7	11

$\frac{11}{7} \neq \frac{7}{4} \neq \frac{4}{1}$

Not exponential – no constant ratio

10)

x	1	2	3	4
y	9	3	1	$\frac{1}{3}$

(·⅓ between each y-value)

$\frac{3}{9} = \frac{1}{3}$ $\frac{1}{3} = \frac{1}{3}$ $\frac{1}{3} \div 1 = \frac{1}{3}$

yes. Exponential decay – constant ratio is $\frac{1}{3}$ (less than 1) It's decreasing.

11)

x	7	8	9	10
y	5	10	20	40

(·2 between each y-value)

$\frac{40}{20} = 2$ $\frac{20}{10} = 2$ $\frac{10}{5} = 2$

yes, exponential growth – y-values are increasing by a constant ratio

Write each formula. (Use your notes)

(Use initial amount a, rate r, time t, principle P, and number of times compounded in a year n)

12) Exponential Growth:

$y = a(1+r)^t$

13) Exponential Decay:

$y = a(1-r)^t$

14) Compound Interest:

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

For problems 15-18, determine whether the function represents *exponential growth* or *decay* (Circle one) and then identify the initial amount (a) and percent rate of change.
Evaluate the function when $t=4$. Round answers to the nearest thousandth. 3 decimal places

15) $y = 3(0.4)^t$

1 − 0.4 = 0.6; 0.4 < 1; .4 is .6 away from 1 or 40% is 60% away from 100%

Growth or Decay?

$a =$ 3

% rate 60%

Evaluate: 0.077

$y = 3(0.4)^4 = 0.0768$

16) $y = 18(1.3)^t$

1.3 is .3 away from 1 (130% is 30% away from 100%); 1.3 > 1

Growth or Decay?

$a =$ 18

% rate 30%

Evaluate: 51.410

$y = 18(1.3)^4 = 51.4098$

17) $y = 41(1.07)^t$

1.07 − 1 = 0.07; 1.07 > 1

Growth or Decay?

$a =$ 41

% rate 7%

Evaluate: 53.743

$y = 41(1.07)^4$

18) $y = 1(0.97)^t$

1 − 0.97 = 0.03; 0.97 < 1

Growth or Decay?

$a =$ 1

% rate 3%

Evaluate: 0.885

$y = (0.97)^4$

For problems 19-21, write a function for the situation given. Then use it to answer the question in part b.

19) Due to an unfortunate zombie apocalypse, a population of 200,000 (a) decreases (decay) by 7.5% (r = 0.075) each year.

a.) Write a function that represents this situation:

$y = 200000(1 - 0.075)^t$

$y = 200000(0.925)^t$

b.) What will the population be after 6 years? ($t = 6$)
(assuming no one figures out how to cure zombies)

$y = 200000(0.925)^6 = 125279.6098$

About 125,279 or 125,280 people

20) A newborn baby dragon weighs 5 pounds (a) and increases (growth) its weight by 25% (r = 0.25) every month until it's fully grown.

a.) Write a function that represents this situation:

$y = 5(1 + 0.25)^t$

$y = 5(1.25)^t$

b.) What will the dragon's weight be after 11 months? ($t = 11$)

$y = 5(1.25)^{11} \approx 58.20766$

About 58.2 pounds

21) Mr. Tibbs invested $6,700 (P) in an account that earns an interest rate of 5.7% (r = 0.057) compounded quarterly. (4 times per year, n = 4)

a.) Write a function that represents the balance after t years:

compound interest: $A = P(1+\frac{r}{n})^{nt}$

$A = 6700(1+\frac{0.057}{4})^{4t}$

$A = 6700(1.01425)^{4t}$

b.) How much money will he have after 10 years? $t=10$

$A = 6700(1.01425)^{4(10)} = 6700(1.01425)^{40}$

$\$11,799.82$

Determine whether the following are *linear* or *exponential.* (Circle one) Then write the function for each. (Hint: When writing the function, use the form $y = mx + b$ for linear and $y = a(b)^x$ for exponential) (#22-25)

(slope, y-int; y-int (initial), ratio (multiplication rate))

22) $(-2,-4), (-1,-1), (0,2), (1,5), (2,8)$

+1, +1, +1; +3, +3, +3, +3

$m = \frac{3}{1} = 3$ y-int (where x = 0) 2

Linear or Exponential ?

Function (equation): $y = 3x + 2$

23)

x	-2	-1	0	1
y	0.25	1	4	16

$\frac{16}{4} = 4$, $\frac{4}{1} = 4$; ·4, ·4, ·4

$a = 4$ $b = 4$

Linear or Exponential ?

Function (equation): $y = 4(4)^x$

24)

x	-2	-1	0	1
y	4	1	-2	-5

+1, +1; −3, −3, −3

$m = \frac{-3}{1} = -3$

Linear or Exponential ?

Function (equation): $y = -3x - 2$

25) $(-2, 25), (-1, 5), (0, 1), \left(1, \frac{1}{5}\right), \left(2, \frac{1}{25}\right)$

$\frac{5}{25} = \frac{1}{5}$, $\frac{1}{5} = \frac{1}{5}$; $\cdot\frac{1}{5}$, $\cdot\frac{1}{5}$, $\cdot\frac{1}{5}$, $\cdot\frac{1}{5}$

$b = \frac{1}{5}$ $a = 1$

$y = 1\left(\frac{1}{5}\right)^x$

Linear or Exponential ?

Function (equation): $y = \left(\frac{1}{5}\right)^x$ or $y = (0.2)^x$

26) Two organizations are collecting donations for a cause. The table shows the numbers of donations collected by Organization A. The graph shows the numbers of donations collected by Organization B.

a.) Compare the organizations by calculating and interpreting the average rates of change from hour 1 to hour 4. (slope from t=1 to t=4)

Organization A:

(1, 4) and (4, 16)

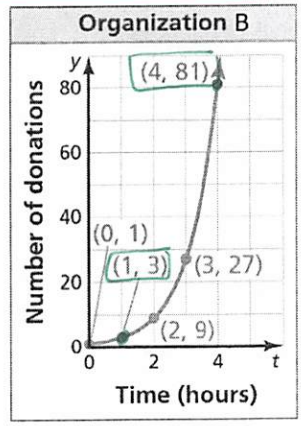
$\frac{16-4}{4-1} = \frac{12}{3}$ 4 donations per hour

Organization B:

(1, 3) and (4, 81) $\frac{81-3}{4-1} = \frac{78}{3}$ 26 donations per hour

(on average, between hours 1 & 4)

Organization A	
Time (hours), t	Number of donations, y
0	0
1	4
2	8
3	12
4	16
5	20
6	24



b.) Predict which organization will have more donations after 24 hours. Explain.

Organization B. They are receiving donations at a much higher rate than organization A.

Solve each equation for x.

27) $2^{x+1} = 2^5$

$x+1=5$

$-1 \quad -1$

$x=4$

28) $3^x = 27$ (3 to what power = 27?)

$3^3 = 27$ (or guess & check)

$x=3$

29) $\frac{1}{49} = 7^x$ ← it became a fraction so the exponent (x) has to be negative

$7^2 = 49$

$x=-2$

Simplify the expression. Your answer should contain only positive exponents.

30) $a^5 \cdot a^7 =$ a^{12}

add the exponents

31) $\frac{(n^4)^3}{n^8}$ multiply exponents

$\frac{n^{12}}{n^8}$ subtract exponents

n^4

32) $\left(\frac{3}{x}\right)^2$ $\frac{3^2}{x^2}$ or $\frac{9}{x^2}$

(or just write it out: $\frac{3}{x} \cdot \frac{3}{x} = \frac{9}{x^2}$)

33) $-3x^2y^4 \cdot 5x^5y^1$ ← no exponent written means exponent is 1

$-3 \cdot 5 = -15$ $x^2 \cdot x^5 = x^7$

$y^4 \cdot y^1 = y^5$ add exponents

$-15x^7y^5$

34) $(2n^5)^3$ multiply exponents

2^3n^{15} or $8n^{15}$

35) $\frac{42a^8c}{6a^5}$ divide subtract exponents

$7a^3c$

36) $6x^{-3}$ move down (or rewrite $(\frac{1}{x})^3$ and go from there)

$\frac{6}{x^3}$

$\frac{6}{1} \cdot \frac{1}{x^3}$

37) $(5x^3)^0 \cdot 2x$

$= 1$

$1 \cdot 2x$

$2x$

38) $\frac{24a^2b^{-7}}{8a^{-4}}$

divide $\frac{24a^2a^4}{8b^7}$

$\frac{3a^6}{b^7}$